



PERGAMON

International Journal of Solids and Structures 38 (2001) 6265–6266

INTERNATIONAL JOURNAL OF  
**SOLIDS and**  
**STRUCTURES**

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## Comments on “A high precision element with a central circular hole” [Int. J. Solids Struct. 36 (1999) 5485–5497]

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The authors Soh and Long (1999) discuss in their paper the construction and use of a special finite element with a circular hole. The functions used satisfy the governing differential equations, and therefore the method utilized can be classified as a Trefftz method (Trefftz, 1926).

Unfortunately, only five references are given in the paper and no reference is made to the literature on the Trefftz method (e.g. Zienkiewicz and Taylor, 1989) and to the special finite elements with holes (Fig. 1) and internal cracks which have been introduced in 1982 (Piltner, 1982, 1985a,b). The special finite elements of reference (Piltner, 1985a) have been cited by several authors (see e.g. Zienkiewicz and Taylor, 1989; Jirousek and Venkatesh, 1992; Kita and Kamiya, 1995; Moorthy and Ghosh, 1996; Pian, 1995; Zhang and Katsebe, 1995; Zielinski, 1995). A brief introduction of the elements with a circular and elliptical hole is available on the internet (Piltner, 1997).

It should be pointed out that in the paper of Soh and Long the problem of coupling a Trefftz-type finite element with a standard displacement element is not mentioned. The coupling method described by the authors is to express the free parameters of the trial functions in terms of the nodal displacements of the element by forcing only element continuity at the element nodes. Constructing an incompatible finite element in this way is considered a variational crime (Strang and Fix, 1973).

In the work of Piltner (1985a) a mixed variational formulation with assumed element boundary displacements is used for the coupling of the special hole element with standard displacement elements. Hole

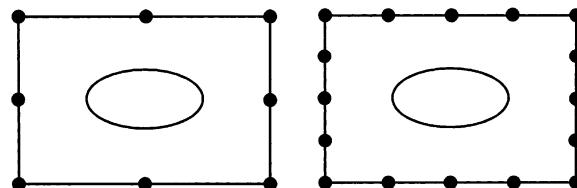


Fig. 1. Special finite elements with holes and piece-wise linear or quadratic boundary displacements (Piltner, 1982, 1985a,b).

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elements with both piecewise linear and quadratic boundary displacements have been introduced in the works of Piltner (1982, 1985a,b), and in addition to the case of a circular hole also elliptical holes and internal cracks have been treated.

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